



**II Semester B.A./B.Sc. Examination, May/June 2018**  
**(CBCS) (F+R) (2014-15 and Onwards)**  
**MATHEMATICS (Paper – II)**

Time : 3 Hours

Max. Marks : 70

**Instruction : Answer all Parts.****PART – A**

1. Answer any five questions : (5×2=10)
- On the set  $Q$ , the set of all rational numbers other than 1,  $*$  is defined by  $a * b = a + b - ab \forall a, b \in Q$ . Find the identity element.
  - Prove that in a group  $G$ ,  $(a^{-1})^{-1} = a \forall a \in G$ .
  - Find the angle between the radius vector and the tangent to the curve  $r = ae^{\theta \cot \alpha}$ .
  - Find the length of the polar subtangent for the curve  $r = a \sec 2\theta$ .
  - Find the asymptotes parallel to the coordinate axes for  $(x^2 + a^2)y = bx^2$ .
  - Find the length of an arc of the cycloid  $x = a(\theta + \sin\theta)$ ,  $y = a(1 - \cos\theta)$ .
  - Show that the equation  $(x^2 - 2xy + 3y^2) dx + (y^2 + 6xy - x^2) dy = 0$  is exact.
  - Solve :  $P^2 - 4P + 3 = 0$  where  $P = \frac{dy}{dx}$ .

**PART – B****Answer one full question :****(1×15=15)**

2. a) Let  $G$  be the set of all rational numbers and  $*$  be the binary operation on  $G$  defined by  $a * b = \frac{ab}{7} \forall a, b \in G$  then prove that  $(G, *)$  is an abelian group. Solve  $4 * x = 3^{-1}$ .
- b) Prove that  $G = \{1, 5, 7, 11\}$  is a group under multiplication modulo 12.
- c) Prove that a non-empty subset  $H$  of a group  $(G, *)$  is a subgroup of  $G$ , if and only if
- $a * b \in H, \forall a, b \in H$
  - $a^{-1} \in H, \forall a \in H$ .

OR



P.T.O.

3. a) Prove that every group of order 4 is abelian.  
 b) Prove that  $G = \{2^n/n \in \mathbb{Z}\}$  is a group under multiplication.  
 c) Prove that  $H = \{1, 2, 4\}$  is a subgroup of the group  $G = \{1, 2, 3, 4, 5, 6\}$  under multiplication modulo 7.

## PART - C

(2×15=30)

Answer any two full questions :

4. a) With usual notations, prove that  $\tan \phi = r \frac{d\theta}{dr}$  for the polar curve,  $r = f(\theta)$ .  
 b) Show that the curves  $r = a(1 + \cos\theta)$ ,  $r = b(1 - \cos\theta)$  intersect orthogonally.  
 c) Show that the evaluate of the parabola  $y^2 = 4ax$  is  $27ay^2 = 4(x - 2a)^3$ .

OR

5. a) Find the angle of intersection of the curves  $r = \sin\theta + \cos\theta$  and  $r = 2 \sin\theta$ .  
 b) Derive the formula for radius of curvature in parametric form.  
 c) Find the pedal equation of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ .
6. a) Find all the asymptotes of the curve  
 $2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$ .  
 b) Find the surface area of the solid generated by revolving about the y-axis the curve  $x = y^3$  from  $y = 0$  to  $y = 2$ .  
 c) Find the position and nature of the double points of the curve  
 $x^3 - y^2 + 4y - 7x^2 + 15x - 13 = 0$ .

OR

7. a) Find the entire length of the asteroid  $x^{2/3} + y^{2/3} = a^{2/3}$ .  
 b) Find the envelope of  $x \cos^3\theta + y \sin^3\theta = c$ , where  $\theta$  is the parameter.  
 c) Find the volume of the solid obtained by revolving the cardioid  $r = a(1 + \cos\theta)$  about the initial line.



PART - D

Answer any one full question :

(1×15=15)

8. a) Solve :  $x \frac{dy}{dx} + (1 - x)y = x^2y^2$ .

b) Solve :  $x = yP + P^2$ .

c) Verify for exactness and solve :

$$(4x + 3y + 1) dx + (3x + 2y + 1) dy = 0.$$

OR

9. a) Solve :  $x \frac{dy}{dx} + y \log y = xye^x$ .

b) Find the general and singular solution of  $y = 3px + 6y^2p^2$ . (Hint-put  $y^3=v$ ).

c) Show that the family of curves  $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$  is self orthogonal,  $\lambda$  is a parameter.

